

Project Report

Optimal Industrial Clusters

Flex4Fact Summer Research Report

Author(s) Madeline Kitch, Mikael Støen, Miguel Muñoz, Thiago Silva, Marc Juanpera, Pau Fisco-Compte

Report Number 2024:00958 — Unrestricted

Client(s) Internal

Technology for a better society

SINTEF Industry

Address: P.O. Box 4760 Torgarden, NO-7465 Trondheim

Telephone: +47 40005100

info@sintef.no Enterprise Number:

919 303 808

KEYWORDS:

Flexible indust Scheduling, Ind Renewables

Project Report

Optimal Industrial Clusters

Flex4Fact Summer Research Report

ABSTRACT

We study the integration of digitization, smart scheduling, local renewable energy production, and variable energy prices in different industrial companies to make an optimization entity for industrial cluster energy use. We discuss two energy optimization approaches: a centralized choice for all energy consumption and local-level trading of energy surplus and develop a graph-based package to implement these mechanisms given mixed-integer programming models for each of the firms. Testing our modeling package with data from the EU project Flex4Fact, we show that clustering decreases aggregate costs due to the lack of sell-back penalties, and the relative benefit among firms depends on internal prices.

Document History

Contents

Chapter 1

Introduction

The electricity system depends on a delicate balance between supply and demand. As storing energy is costly, when expected demand exceeds supply, the grid will often tap into high-price suppliers or shut down end-users entirely. Traditional generation sources such as natural gas and coal are dependable and (relatively) adaptable to demand profiles, reducing the potential scale of this problem. However, as the generation share of variable renewable energies such as solar and wind grows, so will the need for stronger market mechanisms to mitigate potential imbalances.

Flex4Fact (Grant agreement ID: 101058657, DOI: [10.3030/101058657\)](https://doi.org/10.3030/101058657), a project funded by the European Union and coordinated by SINTEF Manufacturing AS, seeks to address how a specific but important group of electricity consumers—industrial manufacturers—can thoughtfully engage in the transitioning energy market. Industrial users consume large amounts of total electricity but have particular challenges in adaptation to energy prices and availability. Since industrial processes are so complex, even formulating a set of options can be difficult, often requiring the digitalization of factory processes. The task of the Flex4Fact is both to work on the development of new market models and methods of engagement between manufacturers and the grid and develop a computational infrastructure that will facilitate their inclusion; to create an "end-to-end ecosystem... to enable flexible manufacturing... in energy intensive industrial sectors" [\[5\]](#page-26-0).

Our work relates to the market design and computational modeling of a subset of this larger market: industrial clusters. Industrial clusters are communities of manufacturers concentrated in a small geographic area. Many industrial firms have started producing energy locally, a phenomenon known as prosuming $[9]$. Prosuming can take the form of on-site Photovoltaic (PV) solar panels, nearby wind turbines, or replacing natural gas with hydrogen [\[4\]](#page-26-0). Yet, the electricity market is presently ill-suited to deal with their supply. As such, prosumers face steep sell-back rates should they have surplus energy, presenting a natural marketplace for energy exchange among firms within a cluster. Firms with extra energy can sell theirs locally for an intermediate rate to others consuming energy, generating a Pareto improvement. Caprara et al. [\[2\]](#page-26-0), a previous Flex4Fact study, documented this Pareto improvement in a case study of rubber production, showing potentially significant energy cost and emissions savings from making a centralized cluster decision.

Inspired by Caprara et al. [\[2\]](#page-26-0), we study and model in detail the problem of local-level energy sharing and optimization industrial clusters, viewing them as a microcosm of the challenges a larger "end-to-end ecosystem" might face—from incentive compatibility constraints to the linkage of digital twins. We formalize the set-up in the language of mechanism design, showing that a perfect (direct) mechanism is unattainable, and, given this, offer two potential approaches: participation-based centralized choice and automatic trading that are simple and transparent. Second, we present a mixed-integer problem formulation and abstract graph-based computational package for both factory-scheduling and energy aggregation suited to the essential needs of clusters. We test this model on Flex4Fact data and show that costs decrease, and the relative benefit of different firms is sensitive to our choice of the internal discount rate or sell-back penalty.

The remainder of this report is structured as follows. Chapter [2](#page-6-0) provides a thorough description of the conceptual framework, business models, and theoretical results. Chapter [3](#page-11-0) presents mixed-integer formula-

tions of factory production scheduling and a market (aggregator) platform. Chapter [4](#page-18-0) presents case studies and their sensitivity to the choice of internal discount rates. Chapter [5](#page-24-0) concludes.

Chapter 2

Conceptual Framework

This chapter discusses the mechanism design set-up for industrial cluster energy sharing.

The high-level structure is as follows: We define firms as the sum of energy management and factory scheduling systems. The energy management (EM) system administrates the energy balance at the factory, taking into account energy flows from their own renewable sources, between other firms in the cluster, and with the grid. The factory scheduling (FS) system optimizes the factory's production profiles based on demand and system constraints. We model the factories individually (business as usual or BAU) and in a centralized cluster. The key difference is that in the cluster case, there is an intermediary entity, the aggregator, between the firms and the grid. The aggregator uses price information and the relative cost of firm's production options to pick the lowest-cost choice for the cluster. A diagram of the two options (individual factories and clusters) are shown in fig. [2.1.](#page-7-0)

To make this problem tractable, a set of simplifying assumptions in line with $[2]$ are needed. First, energy costs are assumed known and linear. This means that firms are able to pick when they want to produce given the prices of the system and that consuming twice as much costs twice as much. Second, it is assumed that different firms have the ability to coordinate energy use profiles. A leading example would be collective weekly scheduling: Each firm would like to make a fixed amount of the good over the period but may present alternative options for when that production would occur. Third, a zero transaction cost or energy loss are assumed when transporting electricity across firms, which is a reasonable assumption considering the low cost and limited loss for short distances.

Firm's Problem. In the simplest model, firms make a fixed amount of goods for a given strategic period but can decide when and how they would like to produce over the shorter intermediate time steps. Each firm also has a local (renewable) energy source which they may use at some cost. Lastly, the firm can decide if they would like to opt into a larger Aggregator system. Below the language used in assignment or allocation problems [\[1\]](#page-26-0) is presented.

Formally, let I be the set of firms (agents) and $\mathrm{J}=\mathbb{R}^T$ the set of possible energy demand vectors to the Aggregator at each of the T operational periods (objects). The notation $\vec{j(i)}=\{j_{i,1},...,\allowbreak j_{i,T}\}$ means that firm i is consuming $j_{i,t}$ energy units from the grid at times $1 \le t \le T$. Furthermore, each firm has a set of energy vectors $S(i)$ for which they are able to meet the product demand. Thus if $\vec{j} \notin S(i)$, demand will not be met. When firms chose among M profiles as in [\[2\]](#page-26-0), $||S(i)|| = M$.

Each firm has preferences π_i over the possible consumption bundles, defined over the set $S(i) \subset J$

$$
\pi_i: \mathrm{J}|_{S(i)} \to \mathbb{R}
$$

These preferences $\pi_i(\vec{j}_i)$ account for the costs independent of non-local energy use or supply. They tell us how different energy consumption profiles cost to implement, excluding those costs associated with acquiring or selling energy through the Aggregator. Examples of components of these preferences include wages to workers, start-up and stopping costs, and material costs.

Figure 2.1: Cluster and benchmark definition

 ${\tt Benchmark}.$ The status-quo environment is modeled as one where firms face price p_t for an energy unit at time t and sell back energy to the grid at a lower price than the grid prices $r \in (0,1)$ where r is the sell-back penalty. Firms pick a profile \vec{j}_i and face total cost C^{BAU}

$$
C^{BAU}(i) = \pi_i(\vec{j}_i) + \sum_{t=1}^T \left[\mathbb{I}_{i,t} j_{i,t} p_t + (1 - \mathbb{I}_{i,t}) r p_t j_{i,t} \right]
$$

where $\mathbb{I}_{i,t}$ an indicator for positive energy demand (i.e. $j_{i,t} > 0$).

Aggregator. The Aggregator is a central entity that can help facilitate the decision of energy use and shared costs. The objective is to minimize the sum of individual objectives while still meeting individual incentivecompatibility constraints. Formally, it can pick options on allocations and subsidies which map the set of firms to their received allotments. We define $\mu\,:\,\rm I\to J$ be the allocation function so that $\mu(i)=\vec{j}_i$, and $t\,:\,\rm I\to\mathbb{R}^{|I|}$ be the subsidy function compensating the losers (those made worse-off by their Aggregator assignments).

Objective. The Aggregator's objective is

$$
TC^{Agg} = \sum_{I} \pi_i(\mu(i)) + \sum_{t=1}^{T} \left[\mathbb{I}_t p_t \sum_{I} \mu(i)_t + (1 - \mathbb{I}_t) r p_t \sum_{I} \mu(i)_t \right]
$$

where \mathbb{I}_t is the indicator for positive net demand at time $t.$ The allocations $\mu({\rm I})$ must satisfy the condition of the firms on being part of the cluster ($t(i)\leq C^{BAU}(i)$) and feasibility for meeting needed production quota $(\vec{j}_i \in J|_{S(i)})$.

2.1 Theoretical results

We present here two theoretical results: (1) clustering energy choice and use will always weakly decrease total costs, and (2) even in a simple case, a direct mechanism cannot exist.

A clustered system, no matter the mechanism, is guaranteed to at least weakly decrease total costs if reporting is truthful. The decrease in cost is due to the fact that internal trading will always save money due to the difference between the purchase and sell-back costs.

Lemma 1. *Under a clustered system, each profile set results in weakly lower costs with inequality when any* two firms would individually buy and sell at the same time. Formally for profile choices $\mu^*(\mathrm{I})=\{j_1,...\,,j_N\},$ $\exists t \in T \text{ and } (i_1, i_2) \in I^2 \text{ s.t. } \mathbb{I}_{i_1, t} \neq \mathbb{I}_{i_2, t}.$

The profile fixed costs are independent of energy aggregation, hence we will focus on the power costs. If all firms buy or sell at time t , then we can factor out the indicators to yield identical costs.

$$
\sum_{I} \pi_i(\vec{j}_i) + \sum_{t=1}^{T} \left[\mathbb{I}_t p_t \sum_{I} j_{i,t} + (1 - \mathbb{I}_t) r p_t \sum_{I} j_{i,t} \right] = \sum_{I} \pi_i(\vec{j}_i) + \sum_{t=1}^{T} \left[\mathbb{I}_t p_t \sum_{I} j_{i,t} + (1 - \mathbb{I}_t) r p_t \sum_{I} j_{i,t} \right]
$$

If not, they differ by

$$
\sum_{\mathbb{I}_{i,t}\neq\mathbb{I}_t} (\mathbb{I}_t - \mathbb{I}_{n,t}) j_{i,t} r p_t - (\mathbb{I}_t - \mathbb{I}_{i,t}) j_{i,t} p_t
$$
\n
$$
= -(1-r) p_t \sum_{\mathbb{I}_{i,t}\neq\mathbb{I}_t} (\mathbb{I}_t - \mathbb{I}_{i,t}) j_{i,t}
$$
\n
$$
= -\text{Sign}(j_{i,t}) (\mathbb{I}_t - \mathbb{I}_{i,t}) (1-r) p_t \sum_{\mathbb{I}_{i,t}\neq\mathbb{I}_{\text{agg}}} |j_{i,t}|
$$
\n
$$
= -(1-r) p_t \sum_{\mathbb{I}_{i,t}\neq\mathbb{I}_t} |j_{n,t}| < 0
$$

Since $r \in (0, 1)$, the Aggregator cost less the benchmark cost is negative; aggregation weakly saves costs for all options and at all time steps.

The second result is that there is unlikely to be a direct mechanism for achieving the optimal outcome, or fairly redistributing the surplus.

Theorem 2. *(Direct Mechanism Impossibility). Assume an alternative profile decreases the cost of one firm and increases the cost of another, and both firms have probabilistic knowledge of the other's preferences, which are continuous on a bounded and overlapping region. Then no truthful, efficient, budget balanced mechanism exists which will always decrease firm costs.*

Our negative result is an exact application of the Meyerson-Satterwaite Theorem where one firm is the "buyer" and the other is a "seller." *Given the impossibility result for a very simple setting, we therefore cannot rule out the fact that firm face incentives to leave out options or artificially alter their preferences should choice be centralized.* Consider the case when factory scheduling happens weekly, and all firms made worse off by the new system will receive subsidies. A firm already knowing their likely allocation may then report higher non-energy costs in order to get additional money from the aggregator. In addition, if they know that another option is preferable for them, they may leave out that energy profile from their option set entirely. *Therefore, even though the Aggregator can always lower costs, the scale of benefit and the fairness of the cost-sharing is critically dependent on the system implementation, the relative preferences of the firms, and the degree of information-sharing.*

2.2 Market Models

The best system design is likely to be a function of the frequency of aggregation, and relative size of the firms. Below we discuss two types of market systems and heuristics as to when both of them may be beneficial.

2.3 Mechanism 1: Automatic Trading

Automatic Trading allows firms to pick their energy consumption and implements a set of automatic trades between users who have demand and surplus in a given period. For instance, if Firm 1 has 10 extra kWh from on-site solar, they could supply it to Firms 2, and 3 will could use 5 kWh each. Firm 1 can now resell at the internal price ($\hat r_{P_t}$) instead of the lower grid one (r_{P_t}). Similarly, Firms 2-3 would face a discounted price equal to the shares and respective prices coming from the internal source and the grid $(\frac{1}{3}\hat{r}p_t + \frac{2}{3})$ $\frac{2}{3}p_t$). The general process is outlined below:

- (1) Firms independently pick their preferred option. Let $\tilde{\mu}(i) \in J$ represent their choice.
- (2) The Aggregator creates new shared buying and selling prices for each of the periods: \hat{p}^{buy}_t \hat{p}^{buy}_t and \hat{p}^{sell}_t .

Define $D(t) = \sum_I \mathbb{I}(\tilde{\mu}(i)_t > 0)\tilde{\mu}(i)_t$ and $S(t) = -\sum_I \mathbb{I}(\tilde{\mu}(i)_t \le 0)\tilde{\mu}(i)_t$ as the total demand from and surplus to the Aggregator at time $t.$ Let $S_{\rm s}(t), S_d(t)$ be the share of internal supply and demand exchanged with the grid. If $S_s(t) = 1$, then at time t, all of the excess supply from the firms is sold to the grid. If $S_d(t) = 1$, all demand is met by the grid.

$$
S_s(t) = \begin{cases} 0 & D(t) > S(t) \\ 1 - \frac{S(t)}{D(t)} & D(t) \le S(t) \end{cases}
$$

$$
S_d(t) = \begin{cases} 0 & D(t) \le S(t) \\ 1 - \frac{S(t)}{D(t)} & D(t) > S(t) \end{cases}
$$

Then the internal prices are given by a weighted combination of the internal and external selling/buying at rate \hat{r} with the general grid prices.

$$
\hat{p}_t^{buy} = \hat{r}(1 - S_d(t))p_t + S_d(t)p_t
$$

and

$$
\hat{p}_t^{sell} = \hat{r}(1 - S_s(t))p_t + S_s(t)r p_t.
$$

(3) Firms now get to face new costs $\tilde{C}(i)$ which are weakly lower than without the trades. As before, $\mathbb{I}_{i,t}$ is the Firm i 's energy buying indicator at time t .

$$
\tilde{C}(i) = \pi_i(\tilde{\mu}(i)) + \sum_T \left[\mathbb{I}_{i,t} \tilde{\mu}(i)_t \hat{p}_t^{buy} + (1 - \mathbb{I}_{i,t}) \tilde{\mu}(i)_t \hat{p}_t^{sell} \right].
$$

Theoretical Advantage

Automatic Trading is preferred when their firms have different objectives. Firms commit to their energy choice prior to trading. As internal trades will only lower cost, the participation constraint will be met. The main issue is that when firms jointly operate at some equilibrium without coordination, they could be far from the global optimum. However, this may be more of a theoretical than a practical issue. For instance, if at least one firm is often consuming a large amount of energy at every time-step, then there is always a demand for local surplus energy. Conversely, if surplus energy of one firm coincides with low or zero electricity demand from all other firms in the cluster, Automatic Trading could be less efficient.

2.4 Mechanism 2: Centralized choice

Centralized choice picks the option that is cost minimizing for the total cluster. After the energy use is set for all firms, internal prices and subsidies determine cost shares.¹

(1) $\mu^*: I \to J$ is defined to minimize system cost below

$$
\sum_{I} \pi_i(\mu(i)) + \sum_{t=1}^{T} \left[\mathbb{I}_t p_t \sum_{I} \mu(i)_t + (1 - \mathbb{I}_t) r p_t \sum_{I} \mu(i)_t \right]
$$

(2) Find the firm's cost of their BAU option

$$
C^{BAU}(i) = \min_{\vec{j} \in J} \pi(\vec{j}) + \sum_{T} \left[\mathbb{I}_{i,t} j_t p_t + (1 - \mathbb{I}_{i,t}) j_t r p_t \right]
$$

- (3) Option 1: Redistribute the costs to align with the BAU case and save the rest for internal cluster investments in energy infrastructure.
- (4) Option 2: Re-pricing
	- Set internal prices (as in Mechanism 1).

$$
\hat{p}_t^{buy} = \hat{r}(1 - S_d(t))p_t + S_d(t)p_t
$$

and

$$
\hat{p}_t^{sell} = \hat{r}(1 - S_s(t))p_t + S_s(t)r p_t.
$$

• Solve for the level of τ needed to create enough revenue to compensate those who are worse off. The losers are given subsidies s_i equal to the difference between their costs under $(\hat{p}_t^{sell},\hat{p}_t^{buy})$ $\int_t^{buy}, r)$ and $C^{BAU}(i)$.

$$
\sum_{t=1}^{T} (1 - \mathbb{I}_t) \left(\sum_{i} \mu(i)_t \right) r p_t = \sum_{i} s_i
$$

Centralized Choice is likely to be advantageous when there is a shared objective or large electricity shock. For instance, if Firms 2-4 are part of the same larger company, then their cumulative goal will be to minimize cost, which is exactly what Mechanism 1 does. When firms have different options, this still may be suitable for one-time events where there is a significant shock to electricity supply. The advantage relative to automatic trading is that more demand options are considered which is especially useful when prices are far from the norm. From a theoretical perspective, the participation constraint is met since firms will be weakly better off as guaranteed by the subsidies. Likewise, all allocations will be feasible. However, centralized choice is not incentive compatible. There is a possibility for untruthful reporting preferences. As energy optimization is repeated for every strategic period, firms may develop knowledge about the likely outcome and the preferences of other firms. A firm, knowing a certain option will be chosen, can make it appear more costly relative to the benchmark (BAU). Alternatively, if they prefer another option, they can leave the optimal one out of their choice-set entirely.

¹The modelling framework presented in this report implements this option without considering subsidies.

PROJECT NUMBER	REPORT NUMBER	VERSION	10 of 26
102027880	2024:00958		

Chapter 3

Problem Formulation of Industrial Clusters

This chapter presents the optimization model that will be used to combine the different factories (considering both decisions in scheduling and energy management) and the grid and aggregator options.

The value-added of this model is two-fold: it both offers a realistic and tractable factory-scheduling input and create a simple avenue for firms to report their preferences. The code for the implementation of the model is written in Julia and designed as a connected set of packages for the modeling tasks. Julia is a fast and dynamic programming language well-suited to solving large optimization systems $[8]$. In order to save and synchronize time steps, we use a package called TimeStruct [\[3\]](#page-26-0). TimeStruct is a SINTEF Julia package that facilitates inputting multi-level time structures into optimization models. In future work this functionality will be used to allow for scheduling between firms that may have different strategic and/or operational periods.

3.1 Factory Scheduling

This model optimizes scheduling of a given factory based on their demand goals, lines and products to be produced. It is originally based on the Polytechnic University of Catalonia (UPC) model for the Standard Profile (SPS) demo case in deliverable D4.2 of Flex4Fact, where the main work has been carried out by the co-authors Marc Juanpera and Pau Fisco-Compte. The model is nevertheless intended to be generalized for other types of factories.

3.1.1 Problem Formulation

The scheduling task consists of creating a production plan for a given factory considering the next temporal horizon T , in these analysis being 5 working days. The factory consists of multiple lines with each having the ability to produce a defined subset of the total set of possible products in the factory. The model should not only consider labour, supplies and product demand but also energy consumption as a tool to reduce the carbon footprint and power costs of the production. The different states have different energy consumption profiles which makes the given approach significantly more complex and realistic. The problem also takes available workers into account. All the relevant data is known beforehand.

The scheduling model formulation is general in that we consider processes that are relevant for many types of production facilities (start-up, switching). Yet it is complex enough to offer relevant and realistic outcomes. Many of the parameters are taken from UPC's work on SPS company data, as mentioned above. Each production line is modelled as a finite state machine with five possible states and combines into a factory. The input data is taken from SPSs case, and are used to ensure relevant and comparable results.

The model is formulated with the following input data and decision variables:

3.1.2 Objective Function

The objective is to minimize the total energy cost ($CONS$) for the manufacturing process, given by the following formula:

Minimize
$$
CONS = \sum_{l \in L} \sum_{p \in P} \left[\sum_{t = \Delta T_p^{ST}}^T pr_{l,p,t} \cdot C_p^E \cdot PR_t^E + \sum_{t' = t \mid t' \leq T}^{T - \Delta T_p^{ST}} tr_{l,p,t} \cdot \sum_{t' = t \mid t' \leq T}^{t + \Delta T_p^{ST}} C_{p,t'-t}^{EST} \cdot PR_{t'}^E + \sum_{t' = t \mid t' \leq T}^{T - \Delta T_{p,p'}^{CT}} \sum_{t' = t \mid t' \leq T}^{T - \Delta T_{p,p'}^{CT}} ch_{l,p,p',t} \cdot \sum_{t' = t \mid t' \leq T}^{t + \Delta T_{p,p'}^{CH}} C_{p,p',t'-t}^{ECH} \cdot PR_{t'}^E \right]
$$
(3.1)

For usage in the aggregator case, the objective function is omitted, and a variable equalling to the consumed energy for each given timestep replaces it, and it is described below in eqs. [\(3.22\)](#page-16-0) to [\(3.25\)](#page-16-0).

3.1.3 Constraints

The feasibility constraint ensures that a product $\,p$ is only produced on a line l if $\,p$ is in the set LP_l :

$$
\sum_{t \in T} pr_{l,p,t} = 0, \quad \forall (l, p) : p \notin LP_l,
$$
\n(3.2)

The demand constraint ensures that the total production of a product p on all lines by its due date is at least equal to its demand:

$$
TC_p \cdot \sum_{t=1}^{TD_p} \sum_{l \in L} pr_{l,p,t} \ge PD_p, \quad \forall p \in P.
$$
 (3.3)

The continuity of the finite state model is ensured by continuity constraints. These constraints ensure that the lines follow the logical progression of the manufacturing process.

Ensuring only one state can be active per line for all t .

$$
\sum_{p \in P} (pr_{p,l,t} + st_{p,l,t} + \sum_{p' \in P|p' \neq p} ch_{p,p',l,t}) + sp_{l,t} + ph_{l,t} \le 1, \quad \forall t \in T, l \in L.
$$
 (3.4)

If line l is stopped at time t , the same line must be either in a stopped of startup state in the next time step. eqs. (3.5) and (3.6) make sure all lines start at either a startup or stopped state.

$$
\sum_{p \in P} st_{p,l,t+1} + sp_{l,t+1} \ge sp_{l,t}, \quad \forall t \in 1 : T-1, l \in L.
$$
 (3.5)

$$
\sum_{p \in P} st_{p,l,1} + sp_{l,1} \ge 1, \quad \forall l \in L.
$$
\n(3.6)

Production constraint, if line l is in a steady production state at time t , the line must be either stopped, changing or continuing production at the next time step

$$
pr_{p,l,t+1} + sp_{l,t+1} + \sum_{p' \in P} ch_{p,p',l,t+1} \ge pr_{p,l,t}, \quad \forall p \in P, l \in L, t \in 1 : T-1
$$
 (3.7)

If line l is starting up production of profile p at time t , the line will be in a production state after the setup time is finished. If line l is starting up at time t , no production can happen before $t+\Delta T_p^{ST}.$ If there is no startup at time t , production can ensue as normal

$$
pr_{p,l,t+\Delta T_p^{ST}} \ge st_{p,l,t}, \quad \forall p \in P, l \in L, t \in 1 : T - \Delta T_p^{ST}
$$
\n
$$
(3.8)
$$

$$
\sum_{t'=t+1}^{t+\Delta T_p^{ST}-1} p h_{l,t} \ge s t_{p,l,t} \cdot \Delta T_p^{ST}-1, \quad \forall p \in P, l \in L, t \in 1 : T - \Delta T_p^{ST}
$$
 (3.9)

If line l is changing production from product \overline{p} to \overline{p}' at time t the line will be in a production state after the changing time is finished. If the line l is changing at time t , no production can happen before $t+\Delta T_P^{CH}$. If there is no change at time t , production can ensue as normal.

$$
pr_{p',l,t+\Delta T_{p,p'}^{CH}} \ge ch_{p,p',l,t}, \quad \forall p, p' \in P \mid p' \ne p, l \in L, t \in 1 : T - \Delta T_{p,p'}^{CH}
$$
\n(3.10)

$$
\sum_{t'=t+1}^{t+\Delta T_{p,p'}^{CH}-1} ph_{l,t} \ge ch_{p,p',l,t} \cdot \Delta T_{p,p'}^{CH} - 1, \quad \forall p, p' \in P, l \in L, t \in I : T - \Delta T_{p,p'}^{CH}
$$
 (3.11)

A line l can not change from product p to the same product.

$$
ch_{p,p,l,t} = 0, \quad \forall p \in P, l \in L, t \in T
$$
\n
$$
(3.12)
$$

A line l can not start up when the remaining time is less or equal to the time required to start up, nor can it change from production of one profile to another when the remaining time is less or equal to the time required to change.

$$
st_{p,l,t} = 0, \quad \forall p \in P, l \in L, t \in T - \Delta T_p^{ST} : T
$$
\n
$$
(3.13)
$$

$$
ch_{p,p',l,t} = 0, \quad \forall p, p' \in P, l \in L, t \in T - \Delta T_{p,p'}^{CH} : T
$$
\n(3.14)

The worker constraint considers available workers at any given time t . The sum of necessary workers for production, startup or change of profile across all lines cannot exceed the total number of available workers at time t .

$$
\sum_{l \in L} \sum_{p \in P} NW_p(pr_{p,l,t} + \sum_{t'=t-\Delta T_p^{ST}+1 \mid t' \ge 0}^{t} st_{p,l,t'} + \sum_{p' \in P} \sum_{t'=t-\Delta T_{p,p'}^{CH}+1 \mid t' \ge 0}^{t} ch_{p,p',l,t'}) \le W_t \quad \forall t \in T
$$
 (3.15)

3.1.4 Implementation

The problem is implemented as a JuMP model in Julia, defined as an internal SINTEF package called *Manufacturing* that it is planned become open source eventually as it is further developed in the project Flex4Fact.

3.1.5 Performance improvements

There are currently two versions of the model. The background for this is the attempt to improve performance by reducing the number of binary variables. The model was successfully reformulated, but early experiments are inconclusive in terms of performance. Therefore, both definitions are presented in this report, although future work will determine which is the best one to be use in the project. Following, the changes needed for the alternative scheduling model's formulation are presented:

- Constraint defined in eq. [\(3.8\)](#page-13-0) and eq. [\(3.10\)](#page-13-0) removed
- Variable $ph_{l,t}$ removed from constraint described in eq. [\(3.4\)](#page-13-0)
- Added constraints eq. (3.16)

Constraints ensuring the time after startup or change has begun has no state until the processes are finished.

$$
\sum_{p' \in P} \sum_{t'=t}^{t+\Delta T_p^{ST}-1} pr_{p',l,t'} + st_{p',l,t'} \le (1 - st_{p,l,t}) \cdot \Delta T_p^{ST}, \quad \forall p \in P, l \in L, t \in 1 : T - \Delta T_p^{ST}
$$
(3.16)

$$
\sum_{p'' \in P} \sum_{t'=t}^{t+\Delta T_{p,p'}^{CH}-1} pr_{p'',l,t'} + st_{p'',l,t'} \le (1-ch_{p,p',l,t}) \cdot \Delta T_{p,p'}^{CH}, \quad \forall p, p' \in P, l \in L, t \in 1 : T - \Delta T_{p,p'}^{CH} \tag{3.17}
$$

3.2 Aggregator Implementation

AggregatorPlatform is an internal SINTEF package, that it is planned become open source eventually as it is further developed in the project Flex4Fact. It takes in factory and energy management models and sends "messages" to send to the market. To facilitate the connection and links between models and the Aggregator we use Plasmo. Plasmo is a package based on JuMP that aggregates different optimization models using an abstract graph structure [\[7\]](#page-26-0). Each model in Plasmo is attached to a node, containing its own set of variables,

Figure 3.1: *Cluster and benchmark definition*

constraints, and objectives. The edges are link constraints, expressions relating variables in different models (nodes). Lastly, groups of nodes can form subgraphs.

When modelling the firms with Plasmo, they are represented by a subgraph with three nodes: a factory scheduling (FS) node, an energy management (EM) node, and a messenger node. We create factory scheduling and energy management JuMP models in separate packages. Alternatively, the user can specify their own JuMP model. These are then attached to their respective nodes and—along with the specified factory name—define a factory structure. The messenger node takes in constraints from the FS and EM models to define the feasible set for each firm. This node, re-labeled with the name of the firm, is the point of contact with the Aggregator. The Aggregator need not have knowledge of the specific inputs or needs given rise to the given domain and preferences. A diagram of the informational flow in the Plasmo model is represented in fig. 3.1.

The node connecting all firms represents the cluster or aggregator, and it is defined as "main". Below the different equations of this node will be presented for its implementation in Plasmo. As the node "main" is an optimization model as well as the other nodes, several variables and constraints are defined.

The power balances at main are defined in eq. (3.18), eq. (3.19) and eq. (3.20).

$$
P_t^{bought,agg} \le M \cdot I_t \quad \forall t \in T \tag{3.18}
$$

$$
P_t^{solid,agg} \le M \cdot (1 - I_t) \quad \forall t \in T
$$
\n
$$
(3.19)
$$

$$
P_t^{bought,agg} - P_t^{solid,agg} = P_t^{agg} \quad \forall t \in T
$$
\n(3.20)

The total power of the aggregator, P_t^{agg} $\mathcal{L}_t^{\text{agg}}$, is based on the power balances of the other energy management and factory nodes, and thus a linking constraint represented in eq. (3.21). The variable $P^{bal}_{n,t}$ is the power balance of each of the energy management and factory nodes:

$$
P_{n,t}^{bal} = P_{n,t}^{EMS} - P_{n,t}^{factory} \quad \forall t \in T, n \in N
$$
\n(3.21)

Decision Variables	Description
P _{bal}	Power balance of each firm n at time t
n,t _D EMS	Power from the energy management system of each firm n at time t
actory	Power demand of the factory for each firm n at time t
√model	Total non-power cost of each model m used for each firm n
$\epsilon_{\mathbf{p}}^{m,n}$	Float, the aggregator's power at time t
C ^{agg}	Float, the aggregator's cost
$\mathcal{C}^{total,agg}$	Float, the aggregator's total cost
$\it c$ power,agg	Float, the aggregator's power cost
	Binary, it defines power sold and bought for the constraints at time t
$P_{i}^{bought,agg}$	Float, bought power by the aggregator at time t
$P_{\cdot}^{sold,agg}$	Float, sold power by the aggregator at time t

Table 3.4: Decision variables related to the aggregator or "main" node

 $P_{n,t}^{factory}$ and $P_{n,t}^{EMS}$ are the power demand at the factory and from the energy management system (for example power from PV or a battery) respectively for firm n and time t . These variables are created in practice in the respective scheduling and energy management system optimization models, and they are linked together at the energy management system node for that factory n , using what Plasmo defines a linking constraint, which is a constraint that involves variables from different optimization models.

 $P_{n,t}^{factory}$ uses in this case the formulation in eq. (3.22) for the case of a scheduling model explained in section section [3.1.](#page-11-0) However, the power demand formulation will depend on the specific scheduling model used for that specific factory.

$$
P_{n_{sch,t}}^{factors} = P_t^{prod} + P_t^{start} + P_t^{change}, \forall t \in T
$$
\n(3.22)

Where n_{sch} represents a factory that models their scheduling using the description in section section [3.1,](#page-11-0) P_t^{prod} where n_{sch} represents a factory that models their scheduling dsing the description in section section 3.1, T_t
is the power consumed by the manufacturing of the products, P_t^{start} is the power consumption caused by star ing producing a product, and $P^{change}_{\bar{t}}$ \mathcal{L}_t^{change} is the power consumption originated when changing from one product to the other in the line, as described in eqs. (3.23) to (3.25) respectively.

$$
P_{t}^{prod} = \sum_{l \in L} \sum_{p \in P} \left[pr_{l,p,t} \cdot C_{p}^{E} \right] \quad t = \Delta T_{p}^{ST}, ..., T
$$
 (3.23)

$$
P_{t+t'-1}^{start} = \sum_{l \in L} \sum_{p \in P} \left[st_{l,p,t} \cdot C_{p,t'-t}^{EST} \right] \quad t = 1, ..., T - \Delta T_p^{ST}, t' = t, ..., t + \Delta T_p^{ST} | t' \le T \tag{3.24}
$$

$$
P_{t+t'-1}^{change} = \sum_{l \in L} \sum_{p \in P} \left[\sum_{p' \in P | p' \neq p} ch_{l,p,p',t} \cdot C_{p,p',t'-t}^{ECH} \right] \quad t = \Delta T_p^{ST}, ..., T - \Delta T_{p,p'}^{CH}, t' = t, ..., t + t + \Delta T_{p,p'}^{CH} | t' \leq T
$$
\n(3.25)

On the other hand, $P_{n,t}^{EMS}$ is the power coming from the EMS. In the case studies it represents a very simple model with a fixed normalized PV profile multiplied by a fixed, installed PV capacity, as shown in eq. (3.26).

$$
P_{n,t}^{EMS} = p_t^{PV} \cdot cap_n^{pv} \quad \forall n \in N, t \in T
$$
\n(3.26)

Where p_t^{PV} is a normalized profile of PV production for time t, which is scaled up with the fixed installed PV capacity cap_n^{pv} at firm n . However, this $P_{n,t}^{EMS}$ can belong to a more complex energy management models,

including batteries, investments etc. Future analyses will use the model [EnergyModelsX](https://github.com/EnergyModelsX) [\[6\]](#page-26-0) for this purpose. Next, the power balance at the aggregator is represented in eq. (3.27), and it is again a liking constraint. As mentioned above each energy management system at a firm n has a variable $P_{n,t}^{bal},$ containing the balance between the energy flows from the energy management system models (e.g. PV produced) and the power demand at the factory, defined in the specific scheduling models. The sum of all the $P_{n,t}^{bal}$ from all firms is linked to the power balance of the aggregator, defined as a variable in the aggregator optimization model in "main".

$$
P_t^{agg} = \sum_{n \in \mathbb{N}} P_{n,t}^{bal} \tag{3.27}
$$

The power costs of the aggregator are calculated as follows, in eq. (3.28).

$$
C^{power,agg} = \sum_{t \in T} P_t^{bought,agg} \cdot Price_t^{power} - P_t^{sold,agg} \cdot Price_t^{power} \cdot r
$$
 (3.28)

The other non-power costs of the aggregator depend on the costs from the energy management and firm nodes, as described in eq. (3.29), and depending on the model use. For the analyses presented in this report, none of these extra costs were considered, but they can represent investments, material costs, maintenance costs, labour costs etc. This is again a linking constraint that connects the different optimization models used through cost variables in this case. \overline{a}

$$
Cagg = \sum_{n \in N, m \in Models} Cm,nmodel
$$
 (3.29)

Finally, the total costs of the aggregator, which equates the objective function of the aggregator consists on the sum of the non-power and power costs, as described in eq. (3.30).

$$
Ctotal,agg = Cagg + Cpower,agg
$$
\n(3.30)

3.3 Practical Challenges

The main challenges from the modelling implementation came from attaching models to JuMP nodes and synchronizing link constraints in TimeStruct in the case of iterating through the time periods. Plasmo does not allow for referencing a node by a string (ie "Firm 1"), meaning that to find the node, we have to search over the labels attached to all nodes in the graph. While tedious from a coding set-up vantage point, there is no performance loss since the messenger nodes must only be found once. In addition, Plasmo doesn't allow for the same model to attach to more than one node. Thus, in constructing graphs, the instance models must be re-defined before assigning them to the FS/EM nodes. Following work in the project has made it easier to navigate through the different nodes and models by among others, using standard naming of nodes and models.

For TimeStruct, it seems important to be careful in referencing time steps as the solving is done for integer time $1 \leq t \leq T$. Using collect (T) or setting an iterator was a useful workaround. A last note is that many of these errors would occur silently. Printing out link constraints, and testing simple cases is therefore very important. Based on these challenges, TimeStruct has been updated to make it easy to iterate through operational periods in future implementations.

Chapter 4

Case studies

This chapter presents a test analysis performed for a generic industrial cluster, with the main purpose of testing the framework and its potential. The main focus will be on the scheduling and the industrial cluster presented in the previous chapter. For each of these models, the setup, main assumptions and results will be presented.

4.1 Main assumptions and data

There are two main parameters that are obtained from real data. The first one is electricity prices, obtained for 120 hours (in the period 3rd-11th of March 2023 and hourly resolution) of Spanish¹ spot prices. For the PV profiles for the different countries, a profile PVGIS² is used. This profile is from a generic PV system in northern Spain, normalized with its maximum capacity. The profile will be then scaled up for each firm that has its own PV system based on the installed capacity. The profiles are shown in fig. [4.1.](#page-19-0) The rest of the values are defined as dummy values.

4.2 Production Scheduling in the Factories

The scheduling test is applied on a single factory, compared to a baseline factory without an optimized schedule. The case regards a factory with 3 products and two lines. The products p1 and p2 can be produced on the same line (l1) while p3 is the only product produced at its own line (l2). p1 and p3 require one worker while p2 requires three. The energy consumption profiles are rather similar for the three products.

There are available workers throughout the entire temporal horizon, but the numbers are reduced for the night shift all week (5 workers on daytime and 3 in the nighttime), so this will effect the production at nighttime since if we produce p2 we are unable to produce anything on the other line. Electricity prices vary with time, following the profile mentioned in section 4.1.

The difference between baseline and smart scheduling is two constraints. For the baseline case each product can only have one startup for the whole period and the model is unable to change between products. One limitation of this approach is that the problem might be infeasible, e.g. if they reduce the number of workers at night, then they might not be able to produce the required product amounts.

4.3 Energy Aggregator for Industrial Cluster

The Aggregation system is tested on a simulated cluster. This cluster is composed of three firms (in the Wizarding World): Honeydukes, Ollivanders, and Hogshead. These three firms have a scheduling model of the one

```
1Downloaded from https://www.ree.es/
```

```
2https://re.jrc.ec.europa.eu/pvg_tools/en/
```


Figure 4.1: Normalized PV profile and electricity prices used for the case studies.

	Honeydukes	Olivanders	Hogshead
Products	p1, p2, p3	p1, p2	p1
Demand per product (units)	1000, 3000, 4000	2000, 2000	2000
Workers needed per product	1, 4, 1	1, 5	
Production per hour and product (units)	50, 50, 50	50, 50	50
Energy use (kWh) per hour and per product	10, 10, 10	10, 10	10
Start-up energy profile (all products, kWh/h)	[20, 30]	[20, 20]	[40, 20]
Lines and products they can produce	11 (p3), 12 (p1, p2)	11 (p1, p2)	11(p1)
Energy profile in each line to change a product (kWh/h)	[20, 20]	[20, 20]	
Installed PV capacity (kW)	O	20	10

Table 4.1: Input data used in the analyses

presented above, in section [3.1](#page-11-0) and section [4.2.](#page-18-0) Honeydukes makes three products on two lines. Line 1 can only make the third product, and line 2 can pick between products 2 and 3. Ollivanders makes two products on 1 line. Hogshead makes one product on one line. They all face a set demand per product which they can meet using the available lines and workers.

They have a fixed amount of time and energy use to begin production. Once in steady-state, they produce a fixed amount of the product with a fixed energy consumption. When switching, they are have set time and energy costs for transitioning between steady states. These switching values can depend on the set of products that are being moved between. In addition, each product requires some number of workers to produce. This is the same for starting, switching and steady-state for a given product. The input data used for the analyses is presented in section [4.3](#page-18-0)

On the energy management side, Honeydukes has no local supply and must always purchase electricity from the grid or the Aggregator. Ollivanders and Hogshead can use their own PV cells. Data for the profile of PV production is scaled from the nominalized profile described in section [4.1](#page-18-0) above depending on the installed capacity of these two firms that has PV.

The optimization will be over one strategic period (120 hours) using electricity spot prices from Spain described in section [4.1.](#page-18-0) Prices peak during times of higher demand (often midday) and generally are decreasing

Figure 4.2: Energy profiles of the case BAU and $r = 0.5$. They represent the energy balance of each firm, also showing a profile with only the factory demand without PV as a dashed line, and the total energy balance of all firms in a black, dotted line.

later in the week.

The analysed cases are built based on the comparison between the Benchmark (individual optimization) and Centralized Choice algorithms. Apart from these two main scenarios, the analysis is complemented by sensitivities to the sell-back discount rate r . In order to calculate the cost for each individual firms energy use we use $\hat{r} = 0.7$ to calculate the internal prices described in [2.](#page-6-0)

4.3.1 Effect of r= 0.5

Recall that when $r = 0.5$ (a value aligning with [\[2\]](#page-26-0)), firms selling energy to the grid by the firms will receive 50% of the price. We discuss this in Section [2.4.](#page-10-0) So the if the price is 2 Euros, they would get 1 Euro. In the Benchmark case for $r = 0.5$, firms do not have enough demand to require continuous production. As such, they are able to save money by postponing production to the end of the week and selling back excess on the earlier days. fig. 4.2 shows both the net energy use from the firms and the energy use relating to the firm's factory. The grey line shows the total power demand to the grid from the Aggregator assuming a Monday - Friday work-week (hours 0 to 120). As seen in fig. [4.1,](#page-19-0) electricity prices decrease for the last days of the week, and this will promote the main part of the production to happen during these days. In addition, there is a considerable amount of PV production except for the second half of the third day. This allows production and energy exchange during the day of most days. We can see these effects of the PV-production and electricity price profiles in the results below.

Under Centralized Choice, shown in fig. [4.3,](#page-21-0) since they can make gains from inter-firm selling we see that in the middle of the week, some factories begin production, boosted by the ability for Ollivanders and Hogshead to supply discounted energy. Suppliers (Ollivanders and Hogshead) can sell surplus energy from PVs at a higher price. Energy users (Honeydukes and Ollivanders) can buy energy at a cheaper rate than later in the week. Like in benchmark case, all firms take advance of the cheap end of week prices for which there is a large aggregate spike in energy demand from the grid.

Figure 4.3: Energy profiles of the Cluster case and $r = 0.5$. They represent the energy balance of each firm, also showing a profile with only the factory demand without PV as a dashed line, and the total energy balance of all firms in a black, dotted line.

Lastly, we plot the difference in benchmark versus cluster costs for the firms in fig. 4.3. These are created by re-defining internal prices for the buying and selling of the energy as discussed in for Centralized Choice. We see here that all firms benefit, and that Honeydukes benefits the most. As we will see, this is a natural result of the relative prices on internal and external exchange. Since the baseline difference between r, \hat{r} is pretty small, the mark-up benefit to the internal sellers is minimal compared to that of internal buyers. This favors the non-producer (Honeydukes).

4.3.2 Effect of r= 0.2

As we raise the external selling discount, this increases the incentive for the energy to be used internally. In the benchmark case this implies that the firm should prioritize use of its PV production. The new benchmark results are plotted in fig. [4.5](#page-22-0) and Cluster results in fig. [4.6.](#page-23-0) As we imagine theoretically, in both cases there is higher demand in the high-price start of the week since the loss from external trading is more costly. Since Honeydukes has no cheap PV energy to use, it still starts at the end of the week. Ollivanders begins much earlier in the week and Hogshead, which has less demand to fulfill, continues to postpone production. In the Cluster case, this result is amplified. Since Honeydukes has cheap energy it can now also take from the other firms, it begins production on Tuesday. Likewise, Ollivanders takes from Hogshead to start on Monday.

As we use post-processing to evaluate cost savings, there is now clear benefit of clustering given to the energy producers (Ollivanders and Hogshead), reversing the earlier trend. The values are presented in fig. [4.7.](#page-23-0)

Figure 4.4: Cost savings for each firm when the cluster case is compared to the benchmark case, r=0.5

Figure 4.5: Energy profiles of the benchmark case and r=0.2. They represent the energy balance of each firm, also showing a profile with only the factory demand without PV as a dashed line, and the total energy balance of all firms in a black, dotted line.

Figure 4.6: Energy profiles of the Cluster case and r=0.2. They represent the energy balance of each firm, also showing a profile with only the factory demand without PV as a dashed line, and the total energy balance of all firms in a black, dotted line.

Figure 4.7: Cost savings for each firm when the cluster case is compared to the BAU case, r=0.2

Chapter 5

Conclusions and next steps

Our project worked to model ways for firms in a cluster to jointly optimize and, in so doing, save costs and best use local energy availability. Going about this required careful attention to how factories made energy-use decisions and how one can link them in a simple computational model. From a market-design side, clustering saves money even if firms pick the same production and renewable energy profiles as before because of the benefit to local selling. Yet, how to pick the best option and share the benefits is an open question. Centralized decision-making has high cost-saving value, but is impractical if firms have different objectives and would like to have control of their production. Automatic trading may offer a promising alternative since it allows firms to operate as they are now with the added bonus of exchanging energy with other firms should they have extra.

The presented modelling framework is suitable for a wide variety of uses. Firms could trade options where they give money to not produce in order to go about factory maintenance. We can also easily adapt the case to include emissions for firms seeking to meet emissions quotas or account for a carbon price. Our next steps on the Aggregator model are as follows:

- Incorporating computationally-tractable ways to share firm information with the aggregator.
- Including emissions into the Aggregation analysis.
- Considering renewable energy investments.

For the production scheduling side, it is important to add more functionalities to model different industrial processes, such as improving workforce constraints (not only number of workers, but also requirements in specialized tasks), more demand fulfilling options or implement material flow, waste and recycling etc.

On the theory side, it is important to better understand concrete cases when industrial energy sharing would happen. Using this, we can offer tailored ideas about how one may properly run a local industrial cluster market. Future analyses will be built on more detailed data from the use cases in Flex4Fact, to actually measure the benefits of a industrial cluster in real life situations.

Beyond the scope of this paper is including an Aggregator in short-term Demand Response (DR) markets. Characteristics depend on the time scale of demand response, but our general advice is to make pricing scheduling and options as transparent as possible to all cluster participants. This will help lessen any efficiency loss in having a two-level (aggregator, DR) approach to energy costs.

Most of all, we hope that our work can be used as a basis for the some of more complex technological and economic questions the Flex4Fact project will seek to address, and others looking at the construction of new business models and digital systems to help facilitate and mitigate the economic impact of the energy transition.

Acknowledgements. We would like to thank Flex4Fact for the funding to explore this interesting question at the intersection of industrial economics, computer science, and the energy transition. In particular, Madeline would like to thank Mikael, Thiago, Miguel and the Industriell økonomi og optimering team at SINTEF for being so welcoming as she studied abroad over the summer, teaching her how to do good collaborative work coding, to use Julia, and the best parts of Norwegian culture.

Bibliography

- [1] Nikhil Agarwal and Eric Budish. 'Market design'. In: *Handbook of Industrial Organization* 5 (1st Jan. 2021), pp. 1–79. DOI: [10.1016/bs.hesind.2021.11.010](https://doi.org/10.1016/bs.hesind.2021.11.010).
- [2] A Caprara et al. 'Industrial Energy Cluster Optimization using Flexibility Aggregation'. In: 17th International Conference on Industrial Engineering and Industrial Management. Barcelona, Spain, July 2023.
- [3] Truls Flatberg and Lars Hellemo. *TimeStruct.jl: Flexible time structures in optimization modelling*. Version 0.7.1. Jan. 2024. DOI: [10 . 5281 / zenodo . 10511399](https://doi.org/10.5281/zenodo.10511399). URL: [https : / / zenodo . org / records /](https://zenodo.org/records/10511399) [10511399](https://zenodo.org/records/10511399) (visited on 20/06/2024).
- [4] Flex4Fact. *D1.1 Industrial Settings Desciption*. 2023.
- [5] *FLEX4FACT project Official website of the Horizon Europe project*. Flex4fact. URL: [https://flex4fact.](https://flex4fact.eu/) [eu/](https://flex4fact.eu/) (visited on 03/04/2024).
- [6] Lars Hellemo et al. 'EnergyModelsX: Flexible Energy Systems Modelling with Multiple Dispatch'. In: *Journal of Open Source Software* 9.97 (2024), p. 6619.
- [7] Jordan Jalving, Yankai Cao and Victor M. Zavala. 'Graph-based modeling and simulation of complex systems'. In: *Computers & Chemical Engineering* 125 (June 2019), pp. 134–154. ISSN: 00981354. DOI: [10 .](https://doi.org/10.1016/j.compchemeng.2019.03.009) [1016/j.compchemeng.2019.03.009](https://doi.org/10.1016/j.compchemeng.2019.03.009). URL: [https://linkinghub.elsevier.com/retrieve/pii/](https://linkinghub.elsevier.com/retrieve/pii/S0098135418312687) [S0098135418312687](https://linkinghub.elsevier.com/retrieve/pii/S0098135418312687) (visited on 10/08/2023).
- [8] *Julia Documentation*. The Julia Language. URL: <https://docs.julialang.org/en/v1/> (visited on 10/08/2023).
- [9] Darja Mihailova. 'Redefining business models for the energy transition: Social innovation and sustainable value creation in the European energy system'. In: 100 (30th May 2023). DOI: $10.1016/j$. erss. 2023. [103114](https://doi.org/10.1016/j.erss.2023.103114).

Technology for a better society